Chapter 17
Appendix A

The Interest Parity Condition

We can derive all the results in the text with a concept that is widely used in international finance. The interest parity condition shows the relationship between domestic interest rates, foreign interest rates, and the expected appreciation of the domestic currency. To derive this condition, we examine how expected returns on domestic and foreign assets are compared.

Comparing Expected Returns on Domestic and Foreign Assets

As in the chapter, we treat the United States as the home country, so domestic assets are denominated in dollars. For simplicity, we use euros to stand for any foreign country’s currency, so we denote foreign assets in euros. To illustrate further, suppose that dollar assets pay a nominal interest rate of $i_D$ and do not have any possible capital gains, so that they have an expected nominal return payable in dollars of $i_D$. Similarly, foreign assets have a nominal interest rate of $i_F$ and an expected nominal return payable in the foreign currency, euros, of $i_F$. To compare the expected returns on dollar assets and foreign assets, investors must convert the returns into the currency unit they use.

We will first examine how François the Foreigner compares the nominal returns on dollar assets and foreign assets denominated in his currency, the euro. When he considers the expected return on dollar assets in terms of euros, he recognizes that it does not equal $i_D$; instead, he must adjust the expected return for any expected appreciation or depreciation of the dollar. If François expects the dollar to appreciate by 3%, for example, the expected return on dollar assets in terms of euros would be 3% higher than $i_D$ because the dollar is expected to become worth 3% more in terms of euros. Thus, if the interest rate on dollar assets is 4%, with an expected 3% appreciation of the dollar, the expected return on dollar assets in terms of euros is 7%; the 4% interest rate plus the 3% expected
appreciation of the dollar. Conversely, if François expects the dollar to depreciate by 3% over the year, the expected return on dollar assets in terms of euros would be only 1%: the 4% interest rate minus the 3% expected depreciation of the dollar.

Writing the current exchange rate (the spot exchange rate) as $E_t$ and the expected exchange rate for the next period as $E^e_{t+1}$, the expected rate of appreciation of the dollar is $[E^e_{t+1} - E_t]/E_t$. Our reasoning indicates that we can write the nominal expected return on dollar assets $R^D$ in terms of foreign currency as the sum of the interest rate on dollar assets plus the expected appreciation of the dollar: 1

$$R^D \text{ in terms of euros} = i^D + \frac{E^e_{t+1} - E_t}{E_t}$$

However, François’s nominal expected return on foreign assets $R^F$ in terms of euros is just $i^F$. Thus, in terms of euros, we calculate the relative expected return on dollar assets (that is, the difference between the expected return on dollar assets and euro assets) by subtracting $i^F$ from the preceding expression to yield

$$\text{Relative } R^D = R^D - i^F + \frac{E^e_{t+1} - E_t}{E_t}$$

(1)

As the relative expected return on dollar assets increases, foreigners will want to hold more dollar assets and fewer foreign assets.

Next let us look at the decision to hold dollar assets versus euro assets from Al the American’s point of view. Following the same reasoning we used to evaluate the decision for François, we know that the nominal expected return on foreign assets $R^F$ in terms of dollars is the interest rate on foreign assets $i^F$ plus the expected appreciation of the foreign currency, equal to minus the expected appreciation of the dollar, $(E^e_{t+1} - E_t)/E_t$.

$$R^F \text{ in terms of dollars} = i^F - \frac{E^e_{t+1} - E_t}{E_t}$$

If the interest rate on euro assets is 5%, for example, and Al expects the dollar to appreciate by 3%, then the nominal expected return on euro assets in terms of dollars

1This expression is actually an approximation of the expected return in terms of euros, which we can calculate more precisely by thinking how a foreigner invests in dollar assets. Suppose François decides to put one euro into dollar assets. First he buys $1/E_t$ of U.S. dollar assets (recall that $E_t$, the exchange rate between dollar and euro assets, is quoted in euros per dollar) and at the end of the period he is paid $(1 + i^D)(1/E_t)$ in dollars. To convert this amount into the number of euros François expects to receive at the end of the period, he multiplies this quantity by the expected return on his initial investment of one euro. His expected return can thus be written as this quantity minus his initial investment of one euro:

$$R^D \text{ in terms of dollars} = \left(1 + i^D\right)\left(\frac{E^e_{t+1}}{E_t}\right) - 1$$

We rewrite this expression as

$$i^D\left(\frac{E^e_{t+1}}{E_t}\right) + \frac{E^e_{t+1} - E_t}{E_t}$$

which is approximately equal to the expression in the text because $E^e_{t+1}/E_t$ is typically close to 1. To illustrate, consider the example in the text in which $(E^e_{t+1} - E_t)/E_t = 0.03$, so $E^e_{t+1}/E_t = 1.03$. Then François’s expected return on dollar assets is $(0.04 \times 1.03) 0.03 = 0.0712 = 7.12\%$, rather than the 7% reported in the text.
is 2%. Al earns the 5% interest rate, but he expects to lose 3% because he expects the euro to be worth 3% less in terms of dollars as a result of the dollar’s appreciation.

Al’s nominal expected return on the dollar assets $R^D$ in terms of dollars is just $i^D$. Hence, in terms of dollars, we calculate the relative expected return on dollar assets by subtracting the expression just given from $i^D$ to obtain

$$\text{Relative } R^D = i^D - \left(i^F - \frac{E^F_{t+1} - E_t}{E_t}\right) = i^D - i^F + \frac{E^F_{t+1} - E_t}{E_t}$$

This equation is the same as Equation 1 describing François’s relative expected return on dollar assets (calculated in terms of euros). The key point here is that the relative expected return on dollar assets is the same—whether it is calculated by François in terms of euros or by Al in terms of dollars. Thus, as the relative expected return on dollar assets increases, both foreigners and domestic residents respond in exactly the same way—both will want to hold more dollar assets and fewer foreign assets.

### Interest Parity Condition

We currently live in a world in which there is capital mobility: foreigners can easily purchase U.S. assets and U.S. investors can easily purchase foreign assets. If there are few impediments to capital mobility and we are looking at assets that have similar risk and liquidity—say, foreign and U.S. bank deposits—then we can reasonably assume the assets are perfect substitutes (that is, equally desirable). When capital is mobile and when assets are perfect substitutes, if the nominal expected return on dollar assets is above that on foreign assets, both foreigners and U.S. investors will want to hold only dollar assets and will be unwilling to hold foreign assets. Conversely, if the nominal expected return on foreign assets is higher than on dollar assets, neither foreign nor U.S. investors will want to hold any dollar assets; they all will want to hold only foreign assets. For investors to hold existing supplies of both dollar assets and foreign assets, it must therefore be true that there is no difference in their nominal expected returns; that is, the relative expected return in Equation 1 must equal zero. We rewrite this condition as,

$$i^D = i^F - \frac{E^F_{t+1} - E_t}{E_t}$$

Equation 2, which is called the **interest parity condition**, states that the domestic interest rate equals the foreign interest rate minus the expected appreciation of the domestic currency. Equivalently, we can state this condition intuitively: the domestic interest rate equals the foreign interest rate plus the expected appreciation of the foreign currency. If the domestic interest rate is higher than the foreign interest rate, there is a positive expected appreciation of the foreign currency, which compensates for the lower foreign interest rate. A domestic interest rate of 5% versus a foreign interest rate of 3% means that the expected appreciation of the foreign currency must be 2% (or, equivalently, that the expected depreciation of the dollar must be 2%).

There are several ways to look at the interest parity condition. First, recognize that interest parity means simply that the nominal expected returns are the same on both dollar assets and foreign assets. To illustrate, note that the left side of the interest parity condition (Equation 2) is the nominal expected return on dollar assets, while the right side is the nominal expected return on foreign assets, both calculated in terms of a sin-
gle currency, the U.S. dollar. Given our assumption that domestic and foreign assets are perfect substitutes (equally desirable), the interest parity condition is an equilibrium condition for the foreign exchange market. Only when the exchange rate is such that nominal expected returns on domestic and foreign assets are equal—that is, when interest parity holds—will investors be willing to hold both domestic and foreign assets.

With some algebraic manipulation, we can rewrite the interest parity condition in Equation 2 as,

\[ E_t = \frac{E_{t+1}^e}{i^F - i^D + 1} \]

Because our assumption of sticky prices implies that real and nominal interest rates move together, this equation produces exactly the same results that we find in the supply and demand analysis in the text: if \( i^D \) rises, the denominator falls and so \( E_t \) rises. If \( i^F \) rises, the denominator rises and so \( E_t \) falls. If \( E_{t+1}^e \) rises, the numerator rises and so \( E_t \) rises.

**KEY TERM**

*interest parity condition, p. 3*

**SUMMARY**

1. The relative expected return on dollar assets is the same—whether it is calculated in terms of euros or in terms of dollars and is as follows:

\[
R^D = i^D - i^F + \frac{E_{t+1}^e - E_t}{E_t}
\]

2. The interest parity condition, Equation 2, \( i^D = i^F - \frac{E_{t+1}^e - E_t}{E_t} \), states that the domestic interest rate equals the foreign interest rate minus the expected appreciation of the domestic currency. Algebraic manipulation of the interest parity condition yields \( E_t = \frac{E_{t+1}^e}{i^F - i^D + 1} \), which produces exactly the same results on what determines the exchange rate as the supply and demand analysis in the chapter. If \( i^D \) rises, the denominator falls and \( E_t \) rises. If \( i^F \) rises, the denominator rises and so \( E_t \) falls. If \( E_{t+1}^e \) rises, the denominator rises and so \( E_t \) rises.
REVIEWS QUESTIONS AND PROBLEMS

1. What is the interest parity condition? Under what circumstances will it hold, and what are its implications?

2. Suppose the interest rate on dollar assets is 2%, the interest rate on Japanese assets is 5%, and the dollar is expected to appreciate by 4% with respect to the Japanese yen in the future.
   a) Calculate the return on dollar assets in terms of the Japanese yen.
   b) What would happen to the exchange rate between the dollar and the Japanese yen? (Hint: what would be the value of the exchange rate for which the interest parity condition holds again?)

3. Suppose the interest rate on dollar-denominated assets is 5% and the interest rate on euro-denominated assets is 9%. What does the interest parity condition imply about the expected appreciation of the dollar with respect to the euro?

4. According to the interest parity condition, what would be the effect of rumors of a future depreciation of the domestic currency on the demand for the domestic asset?

5. According to the interest parity condition, calculate the expected exchange rate between the dollar and the Chinese yuan if the interest rate on dollar-denominated assets is 4%, the interest rate on Chinese yuan-denominated assets is 3%, and the exchange rate between the dollar and the Chinese yuan today is 7 Chinese yuan per dollar.

6. According to the interest parity condition, what would be the effect on the domestic interest rate of a policy that pegs the value of the domestic currency to the value of a foreign currency?