

Chapter 1: Introduction to Functions and Graphs

1.1: Numbers, Data and Problem Solving

Classifying Numbers

1. 400,000 is a real, rational, and natural number. It is also an integer.
3. -13 is an integer, real number, and rational number.
5. 7.5 is a real and rational number.
7. $90\sqrt{2}$ is a real number.
9. Natural number: $\sqrt{9} = 3$; integers: $-3, \sqrt{9}$; rational numbers: $-3, \frac{2}{9}, \sqrt{9}, 1.\bar{3}$; irrational numbers: $\pi, -\sqrt{2}$
11. Shoe sizes are normally measured to within half sizes. Rational numbers are most appropriate.
13. Gasoline is usually measured to a fraction of a gallon using rational numbers.
15. Temperature is typically measured to the nearest degree in a weather forecast. Since temperature can include negative numbers, the integers would be most appropriate.

Real Number Computation

17. Percent change = $\frac{13 - 8}{8} \times 100 = \frac{5}{8} \times 100 = 0.625 \times 100 = 62.5\%$
19. Percent change = $\frac{0.85 - 1.4}{1.4} \times 100 = -0.393 \times 100 = -39.3\%$
21. $187,600 = 1.876 \times 10^5$
23. $0.03983 = 3.983 \times 10^{-2}$
25. $2450 = 2.45 \times 10^3$
27. $0.56 = 5.6 \times 10^{-1}$
29. $10^{-6} = 0.000001$
31. $2 \times 10^8 = 200,000,000$
33. $1.567 \times 10^2 = 156.7$
35. $-5.68 \times 10^{-1} = -0.568$
37. $(9.87 \times 10^6)(34 \times 10^{11}) = 3.3558 \times 10^{19} \approx 3.4 \times 10^{19}$
39. $(56 \times 10^{-7}) + \sqrt{3.4 \times 10^{-2}} \approx 0.184396 \approx 1.8 \times 10^{-1}$
41. $(8.5 \times 10^{-5})(-9.5 \times 10^7) = -8075 = -8.075 \times 10^3$
43. $\sqrt[3]{192} \approx 5.769$
45. $|\pi - 3.2| \approx 0.058$
47. $\frac{(0.3 + 1.5)}{(5.5 - 1.2)} \approx 0.419$
49. $\frac{1.5}{\sqrt{2 + \pi} - 5} \approx -0.549$

$$51. 15 + \frac{4 + \sqrt{3}}{7} \approx 15.819$$

Problem Solving

53. From 1985 to 1998 tuition and fees increased by $\frac{14,508 - 6,121}{6,121} \approx 137.0\%$ while the CPI increased by $\frac{163.0 - 107.6}{107.6} \approx 51.5\%$. The percent increase in the cost of tuition and fees is considerably more than the percent increase in the CPI.
55. The time-and-a-half hourly rate for the seven hours worked over the 40 hours would be $12 + \frac{1}{2}(12) = \$18$. The hourly rate for the 8 hours worked on Sunday would be $2(12) = \$24$. The employee's pay is $40(12) + 7(18) + 8(24) = \798 .
57. The distance Mars travels around the sun is $2\pi r = 2\pi(141,000,000) \approx 885,929,128$ miles. The number of hours in 1.88 years is $365 \times 1.88 \times 24 \approx 16,469$ hours. So Mars' speed is $\frac{885,929,128}{16,469} \approx 53,794$ miles per hour.
59. (a) $\frac{3.7 \times 10^{11}}{2.03 \times 10^8} \approx \$1,820$ per person.
 (b) $\frac{5.54 \times 10^{12}}{2.81 \times 10^8} \approx \$19,715$ per person.
 (c) There are different ways to estimate this.
 1) During the last 30 years the debt per person has increased by $19,715 - 1,820 = 17,895$. Thus, during the next 30 years one might predict the debt per person to be $19,715 + 17,895 = \$37,610$.
 2) During the past 30 years the debt per person has increased by a factor of $\frac{19,715}{1,820} \approx 10.8$. During the next 30 years one might predict the debt to increase by a factor of 10.8 or $10.8 \times \$19,715 = \$212,922$. *Answers may vary.*
61. The area of the film is πr^2 or $\pi(11.5)^2 \approx 415.5$ cm² and the volume of the drop is 0.12 cm³. The thickness of the film is equal to the volume divided by the area. This is, $\frac{0.12}{415.5} \approx 2.9 \times 10^{-4}$ cm.
63. (a) It would take $\frac{5 \times 10^{12}}{100} = 5 \times 10^{10}$ or 50 billion \$100-dollar bills to equal the federal debt. The height of the stacked bills would be $\frac{5 \times 10^{10}}{250} = 2 \times 10^8$ inches or $\frac{2 \times 10^8}{12} \approx 16,666,667$ feet.
 (b) There are 5280 feet in one mile so $\frac{16,666,667}{5250} \approx 3157$ miles. It would reach farther than the distance between Los Angeles and New York.
65. (a) Alcohol consumption on a per capita basis increased each ten-year period between 1940 and 1980. Between 1980 and 1994 alcohol consumption declined.
 (b) There were 199,609,000 people who consumed 2.46 gallons of alcohol on the average. This amounts to $199,609,000 \times 2.46 \approx 4.9 \times 10^8$ gallons of alcohol.
 (c) $\frac{2.46 - 2.76}{2.76} \times 100 = -10.9\%$

1.2: Visualization of Data

Data Involving One Variable

1.

-30	-30	-10	5	15	25	45	55	61
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(a) The maximum is 61 and the minimum is -30.

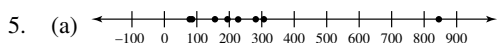
(b) The average is $\frac{-30 - 30 - 10 + 5 + 15 + 25 + 45 + 55 + 61}{9} \approx 15.11$ and the median is 15. The range is $61 - (-30) = 91$.

3. $\sqrt{15} \approx 3.87, 2^{2.3} \approx 4.92, \sqrt[3]{69} \approx 4.102, \pi^2 \approx 9.87, 2^\pi \approx 8.82, 4.1$

$\sqrt{15}$	4.1	$\sqrt[3]{69}$	$2^{2.3}$	2^π	π^2
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(a) The maximum is π^2 and the minimum is $\sqrt{15}$.

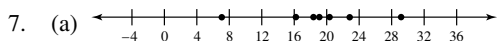
(b) The average is $\frac{\sqrt{15} + 4.1 + \sqrt[3]{69} + 2^{2.3} + 2^\pi + \pi^2}{6} \approx 5.95$ and the median is $\frac{\sqrt[3]{69} + 2^{2.3}}{2} \approx 4.51$. The range is $\pi^2 - \sqrt{15} \approx 6.00$.



(b) Average = $\frac{840 + 87.8 + 227 + 280 + 84.2 + 196 + 306 + 165}{8} = 273.25 \approx 273$;

Median = $\frac{196 + 227}{2} = 211.5 \approx 212$; Range = $840 - 84.2 = 755.8 \approx 756$. The average area of the eight largest islands in the world is 273,000 square miles. Half of the islands have areas below 212,000 square miles and half have areas above 212,000 square miles. The largest difference in area between any two islands is 756,000.

(c) The largest island is Greenland with 840,000 square miles.



(b) Average = $\frac{19.3 + 18.5 + 29.0 + 7.31 + 16.1 + 22.8 + 20.3}{7} \approx 19.0$; Median = 19.3;

Range = $29.0 - 7.31 \approx 21.7$. The average of the maximum elevations of the seven continents is 19,000 feet. About half of these continents have maximum elevations below 19,300 feet and about half are above. The largest difference between these elevations is 21,700 feet.

(c) The mountain with the highest elevation is Mount Everest in Asia.

9. (a) Average = $\frac{385,000 + 385,000 + 385,000 + 385,000 + 385,000 + 14,795,642}{6} \approx \$2,786,774$

(b) Median = $\frac{385,000 + 385,000}{2} = \$385,000$

(c) One large salary can raise the average considerably, while the majority of the players have considerably lower salaries.

11. *Answers may vary.* 16, 18, 26; No

Distance Formula

13. $d = \sqrt{(5 - 2)^2 + (2 - (-2))^2} = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$

15. $d = \sqrt{(9 - 7)^2 + (1 - (-4))^2} = \sqrt{2^2 + 5^2} = \sqrt{29} \approx 5.39$

17. $d = \sqrt{(3.6 - (-6.5))^2 + (-2.9 - 2.7)^2} = \sqrt{10.1^2 + (-5.6)^2} = \sqrt{133.37} \approx 11.55$

19. $d = \sqrt{(-3 - (-3))^2 + (10 - 2)^2} = \sqrt{0^2 + 8^2} = \sqrt{64} = 8$

21. $d = \sqrt{\left(\frac{3}{4} - \frac{1}{2}\right)^2 + \left(\frac{1}{2} - \left(-\frac{1}{2}\right)\right)^2} = \sqrt{\left(\frac{1}{4}\right)^2 + 1^2} = \sqrt{\frac{1}{16} + 1} = \sqrt{\frac{17}{16}} = \frac{\sqrt{17}}{4} \approx 1.03$

23. $d = \sqrt{(a - a)^2 + (a - b)^2} = \sqrt{0 + (a - b)^2} = \sqrt{(a - b)^2} = |a - b|$

25. $d = \sqrt{(3 - 0)^2 + (4 - 0)^2} = \sqrt{4^2 + 3^2} = \sqrt{16 + 9} = \sqrt{25} = 5$

$$d = \sqrt{(7 - 3)^2 + (1 - 4)^2} = \sqrt{4^2 + (-3)^2} = \sqrt{16 + 9} = \sqrt{25} = 5$$

The side between (0,0) and (3,4) and the side between (3,4) and (7,1) have equal length, so the triangle is isosceles.

27. (a) See Figure 27

(b) $d = \sqrt{(0 - (-40))^2 + (50 - 0)^2} = \sqrt{40^2 + 50^2} = \sqrt{1600 + 2500} = \sqrt{4100} \approx 64.0$ miles.

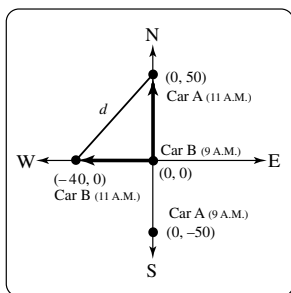


Figure 27

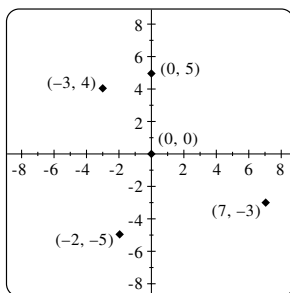


Figure 29

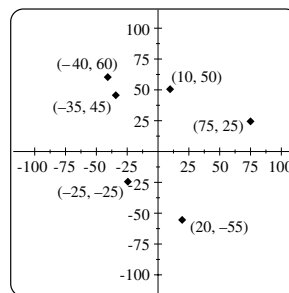


Figure 31

Data Involving Two Variables

29. (a) The domain is $D = \{0, -3, -2, 7\}$ and the range is $R = \{5, 4, -5, -3, 0\}$

(b) The minimum x -value is -3 , and the maximum x -value is 7 . The minimum y -value is -5 , and the maximum y -value is 5 .

(c) The axes must include at least $-3 \leq x \leq 7$ and $-5 \leq y \leq 5$. It would be appropriate to extend each axis slightly beyond these intervals and let each tick mark represent 1 unit. Plot the points $(0, 5)$, $(-3, 4)$, $(-2, -5)$, $(7, -3)$, and $(0, 0)$.

(d) See Figure 29.

31. (a) The domain is $D = \{10, -35, 20, 75, -40, -25\}$ and the range is $R = \{50, 45, -55, 25, 60, -25\}$
 (b) The minimum x -value is -40 , and the maximum x -value is 75 . The minimum y -value is -55 , and the maximum y -value is 60 .
 (c) The axes must include at least $-40 \leq x \leq 75$ and $-55 \leq y \leq 60$. It would be appropriate to extend each axis slightly beyond these intervals. Plot the points $(10, 50)$, $(-35, 45)$, $(20, -55)$, $(75, 25)$, $(-40, 60)$, and $(-25, -25)$.
 (d) See Figure 31.
33. (a) The domain is $D = \{0.1, 0.5, -0.7, 0.8, 0.9\}$ and the range is $R = \{-0.3, 0.4, 0.5, -0.1, 0.9\}$
 (b) The minimum x -value is -0.7 , and the maximum x -value is 0.9 . The minimum y -value is -0.3 , and the maximum y -value is 0.9 .
 (c) The axes must include at least $-0.7 \leq x \leq 0.9$ and $-0.3 \leq y \leq 0.9$. It would be appropriate to extend each axis slightly beyond these intervals. Plot the points $(0.1, -0.3)$, $(0.5, 0.4)$, $(-0.7, 0.5)$, $(0.8, -0.1)$, and $(0.9, 0.9)$.
 (d) See Figure 33.

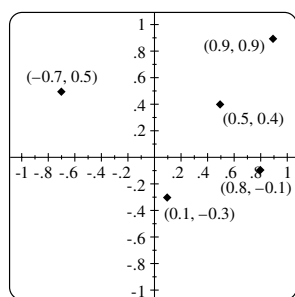


Figure 33

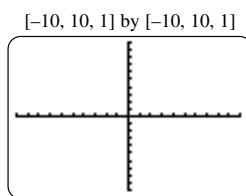


Figure 35

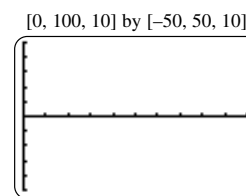


Figure 37

35. x -axis: 10 tick marks; y -axis: 10 tick marks. See Figure 35.
 37. x -axis: 10 tick marks; y -axis: 5 tick marks. See Figure 37.
 39. x -axis: 16 tick marks; y -axis: 5 tick marks. See Figure 39.

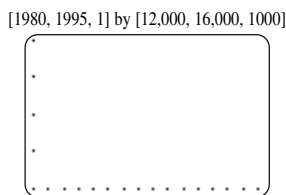


Figure 39

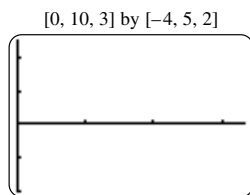


Figure 41

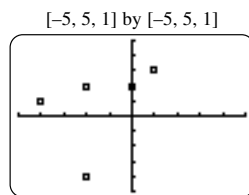


Figure 47

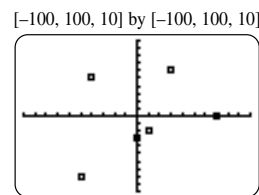


Figure 49

41. x -axis: 3 tick marks; y -axis: 2 tick marks. See Figure 41.
 43. Graph (b)
 45. Graph (a)
 47. Plot the points $(1, 3)$, $(-2, 2)$, $(-4, 1)$, $(-2, -4)$ and $(0, 2)$ in $[-5, 5, 1]$ by $[-5, 5, 1]$, See Figure 47.
 49. Plot the points $(10, 20)$, $(-40, 50)$, $(30, 60)$, $(-50, 80)$, $(70, 0)$, and $(0, -30)$ in $[-100, 100, 10]$ by $[-100, 100, 10]$, See Figure 49.

51. Plot the points $(3.1, 6.2)$, $(-5.1, 10.1)$, $(-0.7, -1.4)$, $(1.8, 3.6)$ and $(-4.9, -9.8)$ in $[-6, 6, 2]$ by $[-12, 12, 2]$,

See Figure 51.

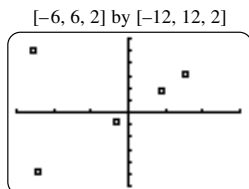


Figure 51

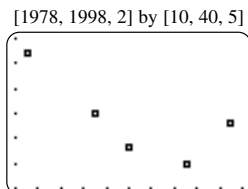


Figure 53a

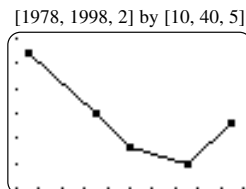


Figure 53b

53. (a) x -min: 1979; x -max: 1997; y -min: 15; y -max: 37

(b) $[1978, 1998, 2]$ by $[10, 40, 5]$. *Answers may vary.*

(c) Plot the points $(1979, 37)$, $(1985, 25)$, $(1988, 18)$, $(1993, 15)$ and $(1997, 23)$ in $[1978, 1998, 2]$ by $[10, 40, 5]$,

See Figure 53a.

(d) See Figure 53b.

55. (a) x -min: 1996; x -max: 2004; y -min: 9.7; y -max: 12.8

(b) $[1994, 2006, 1]$ by $[8, 14, 1]$. *Answers may vary.*

(c) Plot the points $(1996, 9.7)$, $(1998, 10.5)$, $(2000, 11.2)$, $(2002, 12.0)$ and $(2004, 12.8)$ in

$[1994, 2006, 1]$ by $[8, 14, 1]$, See Figure 55a.

(d) See Figure 55b.

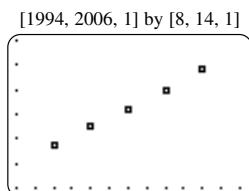


Figure 55a

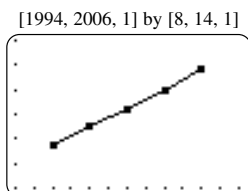


Figure 55b

57. Number of doctorate degrees conferred in the United States. *Answers may vary.*

Year	1950	1960	1970	1980	1990	1995
Doctorate Degrees	6000	10,000	30,000	33,000	38,000	41,000

1.3: Functions and Their Representations

Evaluating and Representing Functions

1. If $f(-2) = 3$, then the point $(-2, 3)$ is on the graph of f .
3. If $(7, 8)$ is on the graph of f , then $f(7) = 8$.
5. See Figure 5.
7. See Figure 7.

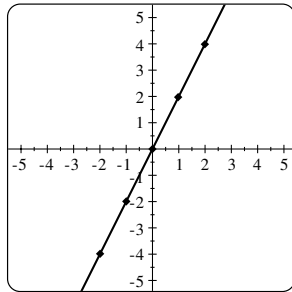


Figure 5

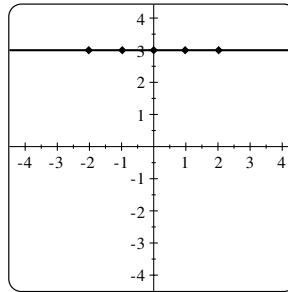


Figure 7

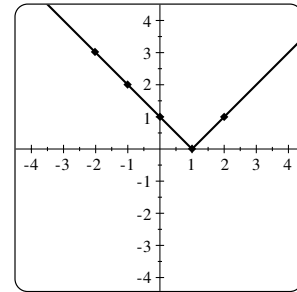


Figure 9

9. See Figure 9.

11. See Figure 11.

13. See Figure 13

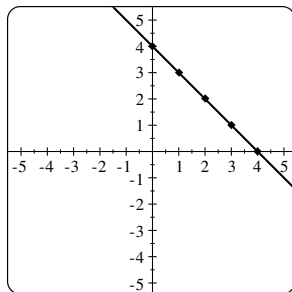


Figure 11

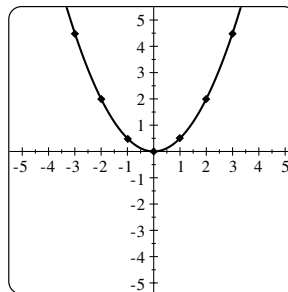


Figure 13.

15. (a) $f(x) = x^3 \Rightarrow f(-2) = (-2)^3 = -8$ and $f(5) = 5^3 = 125$.

(b) The domain of f includes all real numbers.

17. (a) $f(x) = \sqrt{x} \Rightarrow f(-1) = \sqrt{-1}$ which is not a real number, and $f(a + 1) = \sqrt{a + 1}$.

(b) The domain of f includes all nonnegative real numbers.

19. (a) $f(x) = \frac{1}{x-1} \Rightarrow f(-1) = \frac{1}{-1-1} = -\frac{1}{2}$ and $f(a+1) = \frac{1}{(a+1)-1} = \frac{1}{a}$.

(b) The domain of f includes all real numbers not equal to 1 ($x \neq 1$).

21. (a) $f(x) = -7 \Rightarrow f(6) = -7$ and $f(a-1) = -7$. (f is a constant function.)

(b) The domain of f includes all real numbers.

23. (a) $f(x) = \frac{1}{x^2} \Rightarrow f(4) = \frac{1}{4^2} = \frac{1}{16}$ and $f(-7) = \frac{1}{(-7)^2} = \frac{1}{49}$.

(b) The domain of f includes all real numbers not equal to 0 ($x \neq 0$).

25. (a) $f(x) = \frac{1}{x^2 - 9} \Rightarrow f(4) = \frac{1}{4^2 - 9} = \frac{1}{16 - 9} = \frac{1}{7}$ and

$$f(a-5) = \frac{1}{(a-5)^2 - 9} = \frac{1}{a^2 - 10a + 25 - 9} = \frac{1}{a^2 - 10a + 16}.$$

(b) The domain of f includes all real numbers not equal to 3 or -3 ($x \neq 3, x \neq -3$).

27. The domain of f includes all real numbers x such that $-3 \leq x \leq 3$.

The range of f includes all real numbers y such that $0 \leq y \leq 3$.

$$f(0) = 3.$$

29. The domain of f includes all real numbers x such that $-2 \leq x \leq 4$.

The range of f includes all real numbers y such that $-2 \leq y \leq 2$.

$$f(0) = -2.$$

31. The domain of f includes all real numbers x .

The range of f includes all real numbers y such that $y \leq 2$.

$$f(0) = 2.$$

33. The domain of f includes all real numbers x such that $x \geq -1$.

The range of f includes all real numbers y such that $y \leq 2$.

$$f(0) = 0.$$

35. (a) Since the point $(0, -2)$ lies on the graph of f , $f(0) = -2$. Similarly, $f(2) = 2$.

(b) We must find all points (x, y) on the graph of f where $y = 0$. There is only one point where this occurs:

$$(1, 0). \text{ Thus, when } x = 1, f(x) = 0.$$

37. (a) Since the point $(0, 0)$ lies on the graph of f , $f(0) = 0$. Similarly, $f(2) = 4$.

(b) We must find all points (x, y) on the graph of f where $y = 0$. There is only one point where this occurs:

$$(0, 0). \text{ Thus, when } x = 0, f(x) = 0.$$

39. (a) Since the point $(0, 0)$ lies on the graph of f , $f(0) = 0$. Similarly, $f(2) = 0$.

(b) We must find all points (x, y) on the graph of f where $y = 0$. There are three points where this occurs:

$$(-2, 0), (0, 0) \text{ and } (2, 0). \text{ Thus, when } x = -2, 0, \text{ or } 2, f(x) = 0.$$

41. Graph $f(x) = 0.25x^2$ in $[-4.7, 4.7, 1]$ by $[-3.1, 3.1, 1]$ by letting $Y_1 = 0.25X^2$. See Figure 41.

(a) From the graph it appears that $f(2) \approx 1$.

$$(b) \text{ Evaluating } f(2) = 0.25(2)^2 = 0.25(4) = 1.$$

43. Graph $f(x) = \sqrt{x+2}$ in $[-4.7, 4.7, 1]$ by $[-3.1, 3.1, 1]$ by letting $Y_1 = \sqrt{X+2}$. See Figure 43.

(a) From the graph it appears that $f(2) = 2$.

$$(b) \text{ Evaluating } f(2) = \sqrt{2+2} = \sqrt{4} = 2.$$

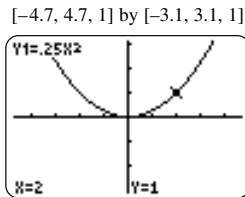


Figure 41

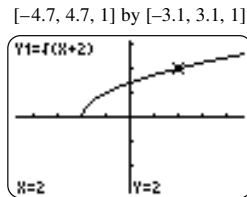


Figure 43

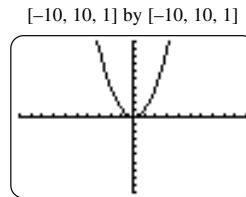


Figure 45

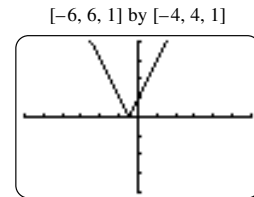


Figure 47

45. Verbal: Square the input x .

Graphical: Graph $Y_1 = X^2$. See Figure 45.

Numerical:

-2	-1	0	1	2
4	1	0	1	4

47. Verbal: Multiply the input x by 2, add 1, and then take the absolute value.

Graphical: Graph $Y_1 = \text{abs}(2X + 1)$. See Figure 47.

Numerical:

-2	-1	0	1	2
3	1	1	3	5

49. Verbal: Compute the absolute value of x .

Graphical: Graph $Y_1 = \text{abs}(X)$. See Figure 49.

Numerical:

-2	-1	0	1	2
2	1	0	1	2

51. Verbal: Add 1 to the input x and then take the square root of the result.

Graphical: Graph $Y_1 = \sqrt{X + 1}$. See Figure 51.

Numerical:

-2	-1	0	1	2
Error	0	1	$\sqrt{2}$	$\sqrt{3}$

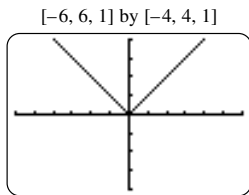


Figure 49

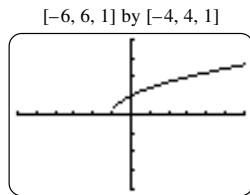


Figure 51

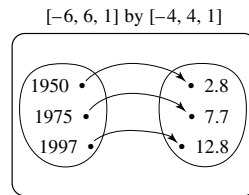


Figure 55

53.

Bills (millions)	0	1	2	3	4	5	6
Counterfeit Bills	0	9	18	27	36	45	54

55. (a) See Figure 55

(b) $f(1975) = 7.7$; in 1975 there were 7700 radio stations on the air.

(c) $D = \{1950, 1975, 1997\}$ and $R = \{2.8, 7.7, 12.8\}$.

Identifying Functions

57. This is a graph of a function because every vertical line intersects the graph at most once. Both the domain and the range are all real numbers..

59. This is not a graph of a function because some vertical lines can intersect the graph at twice. Because a vertical line can intersect the graph twice, two functions are necessary to create this graph.
61. This is a graph of a function because every vertical line intersects the graph at most once.
The domain is $-4 \leq x \leq 4$. The range is $0 \leq y \leq 4$.
63. Yes. The calculation of the cube root requires the input of a real number. It produces a single output.
A number has only one cube root.
65. No. On most English exams, more than one person passes. Therefore, for a given English exam as input, there is more than one output. A listing does not typically compute an output.
67. No. The ordered pairs $(1, 2)$ and $(1, 3)$ belong to the set S . The domain element 1 has more than one range element associated with it.
69. Yes. Each element in its domain is associated with exactly one range element.
71. No. The ordered pair $(1, 10.5)$ and $(1, -0.5)$ belong to the set S . The domain element 1 has more than one range element associated with it.
73. No. The ordered pair $(1, -1)$ and $(1, 1)$ belong to the function f . The domain element 1 has more than one range element associated with it.
75. Yes. Each element in the domain of f is associated with exactly one range element.

Applications

77. Let x = time in seconds that elapsed for the observer to hear the thunder from the lightning bolt. The distance is found by multiplying the speed by the time. Since $5280 \text{ ft.} = 1 \text{ mi.}$, the distance in miles is given by

$$f(x) = \frac{1150}{5280}x = \frac{115}{528}x; f(15) = \frac{115}{528}(15) \approx 3.3;$$
 with a 15-second delay, the lightning bolt was about 3.3 miles away.

79. Verbal: Multiply the input x by -5.8 to obtain the change in temperature.

Symbolic: $f(x) = -5.8x$

Graphical: $Y_1 = -5.8X$. See Figure 79a.

Numerical: Table $Y_1 = -5.8X$. See Figure 79b.

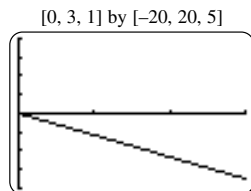


Figure 79a

X	Y ₁
0	0
.5	-2.9
1	-5.8
1.5	-8.7
2	-11.6
2.5	-14.5
3	-17.4

Y₁ = -5.8X

Figure 79b

1.4: Types of Functions and Their Rates of Change

Slope

1. $m = \frac{5 - 6}{2 - 4} = 0.5$
3. $m = \frac{7.6 - 9.2}{-0.3 - (-0.5)} = -8$
5. $m = \frac{7.9 - 5.6}{1994 - 1997} = -\frac{23}{30} \approx -0.7667$
7. $m = \frac{8 - 6}{-5 - (-5)} = \text{undefined}$
9. $m = \frac{\frac{7}{10} - (-\frac{3}{5})}{-\frac{5}{6} - \frac{1}{3}} = \frac{\frac{13}{10}}{-\frac{7}{6}} = \frac{13}{10} \cdot \left(-\frac{6}{7}\right) = -\frac{39}{35}$
11. Slope = 2; the graph rises 2 units for every unit increase in x .
13. Slope = $-\frac{3}{4}$; the graph falls $\frac{3}{4}$ unit for every unit increase in x ; or equivalently, the graph falls 3 units for every 4-unit increase in x .
15. Slope = 0; the graph neither falls nor rises for every unit increase in x since the y -value is always -5 .
17. (a) To find the median age in 1980 and 2000, we must evaluate $f(1980)$ and $f(2000)$.
 $f(1980) = 0.243(1980) - 450.8 = 30.34$ and $f(2000) = 0.243(2000) - 450.8 = 35.2$
 In 1980, the median age was 30.34 years and in 2000, it increased to 35.2 years.
- (b) Since $f(x) = 0.243x - 450.8$, the slope of its graph is $m = 0.243$. The value of 0.243 means that the median age in the United States has increased by approximately 0.243 for each year from 1980 to 2000.
19. (a) See Figure 19.
- (b) Since $f(x) = -0.0635x + 147.9$, the slope of its graph is $m = -0.0635$. A slope of $m = -0.0635$ means that the time to run the 200-meter dash has decreased by approximately 0.0635 seconds per year.
- (c) In a four-year period, times have changed by approximately $4(-0.0635) = -0.254$ seconds.

[1948, 1996, 4] by [21, 25, 1]

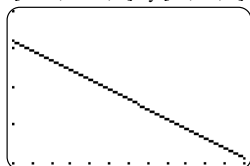


Figure 19

[-10, 10, 1] by [-10, 10, 1]

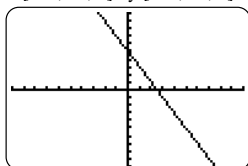


Figure 21

[-10, 10, 1] by [-10, 10, 1]

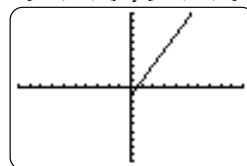


Figure 23

[-10, 10, 1] by [-10, 10, 1]

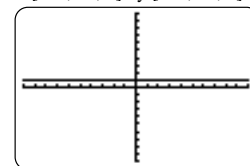


Figure 25

Linear and Nonlinear Functions

21. $f(x) = -2x + 5$ is a linear function with a slope of $m = -2$. See Figure 21.
23. $f(x) = 2x^{1.01} - 1$ is a nonlinear function because the exponent on x is not equal to one. See Figure 23.
25. $f(x) = 1$ is a constant (and linear) function. See Figure 25.

27. From its graph, we see that $f(x) = 0.2x^3 - 2x + 1$ represents a nonlinear function. See Figure 27.

29. From its graph, we see that $f(x) = 5x - x^2$ represents a nonlinear function. See Figure 29.

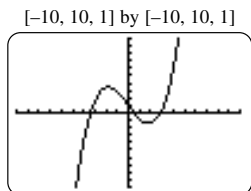


Figure 27

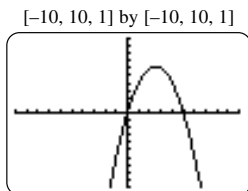


Figure 29

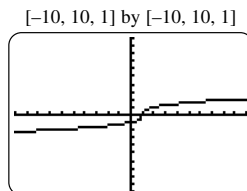


Figure 31

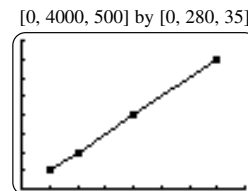


Figure 39

31. From its graph, we see that $f(x) = \sqrt[3]{x-1}$ represents a nonlinear function. See Figure 31.

33. Between each pair of points, the y -values increase 4 units for each unit increase in x . Therefore, the data is linear. The slope of the line passing through the data points is 4.

35. The y -values do not increase by a constant amount for each 2-unit increase in x . The data is nonlinear.

37. The y -values decrease 0.5 units for each unit increase in x . Therefore, the data is linear. The slope of the line passing through the data points is -0.5 .

39. (a) See Figure 39.

(b) From the line graph it can be seen that f is linear. The slopes of the line segments are all equal.

41. (a) See Figure 41.

(b) From the line graph it can be seen that the data is not linear. The function f is nonlinear.

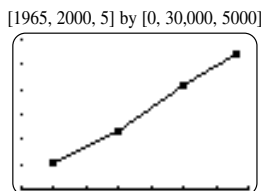


Figure 41

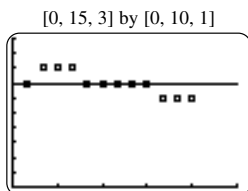


Figure 43

43. (a) Since the wind speed is not constant, it cannot be represented exactly by a constant function.

(b) The average of the twelve wind speeds is 7 miles per our. It would be reasonable to approximate the wind speed at Myrtle Beach by $f(x) = 7$.

(c) A scatterplot of the data and the function $f(x) = 7$ are shown in Figure 43.

45. (a) None of the functions are constant. f is linear since it increases by 2π for each unit increase in x . Similarly, h is also linear because it increases 2 units for each unit increase in x . The function g is nonlinear since it does not increase by a constant amount for each unit increase in x .

(b) f computes the circumference: $2\pi x$, g computes the area: πx^2 and h computes the diameter: $2x$.

47. If the distance is constant, then the car is not moving. The car's velocity is zero.

49. $f(x) = \frac{x}{16}$

51. $f(x) = 50x$ (miles)

53. $f(x) = 500$

55. $f(x) = 6x + 1$

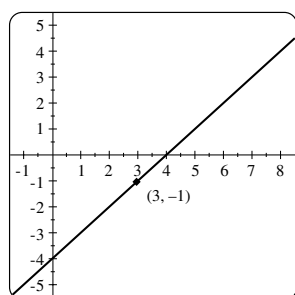
Curve Sketching57. Sketch a line that is sloping upward from left to right. The line must pass through the point $(3, -1)$.See Figure 57. *Answers may vary.*59. See Figure 59. *Answers may vary.*61. See Figure 61. *Answers may vary.*

Figure 57

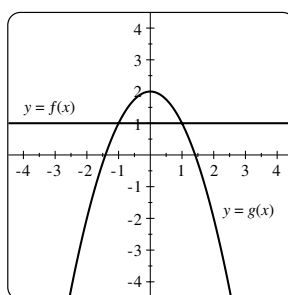


Figure 59

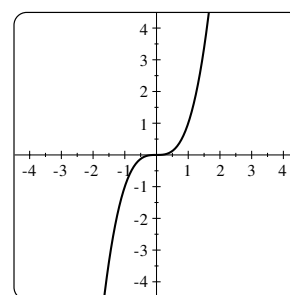


Figure 61

Average Rates of Change

63. The average rate of change from -3 to -1 is $\frac{f(-1) - f(-3)}{-1 - (-3)} = \frac{4 - 4}{2} = 0$.

The average rate of change from 1 to 3 is $\frac{f(3) - f(1)}{3 - 1} = \frac{4 - 4}{2} = 0$.

65. The average rate of change from -3 to -1 is $\frac{f(-1) - f(-3)}{-1 - (-3)} = \frac{3.7 - 1.3}{2} = \frac{2.4}{2} = 1.2$.

The average rate of change from 1 to 3 is $\frac{f(3) - f(1)}{3 - 1} = \frac{1.3 - 3.7}{2} = -\frac{2.4}{2} = -1.2$.

67. If $f(x) = 7x - 2$ then $\frac{f(4) - f(1)}{4 - 1} = 7$. The slope of the graph is 7.

69. If $f(x) = x^3 - 2x$ then $\frac{f(4.5) - f(0.5)}{4.5 - 0.5} = 20.75$. The slope of the line passing through the points $(0.5, f(0.5))$ and $(4.5, f(4.5))$ is 20.75.

71. If $f(x) = \sqrt{2x - 1}$ then $\frac{f(3) - f(1)}{3 - 1} \approx 0.6180$. The slope of the line passing through the points $(1, f(1))$ and $(3, f(3))$ is approximately 0.6180.

73. (a) Make a line graph using the ordered pairs (x, E) for the data points. See Figure 73.
- (b) From 0 to 10 the average rate of change is $\frac{f(10) - f(0)}{10 - 0} = \frac{69.9 - 72.3}{10} = -0.24$. Similarly, the average rate of change is -0.98 from 10 to 20, -0.97 from 20 to 30, -0.95 from 30 to 40, -0.93 from 40 to 50, -0.85 from 50 to 60, -0.76 from 60 to 70, and -0.63 from 70 to 80. The average rate of change is always negative. As a woman becomes older, her *remaining* life expectancy decreases. For example, if the rate of change is -0.63 , then for each year a woman lives, her remaining life expectancy decreases by 0.63 years.
- (c) A person 20 years old can expect to live an additional 60.1 years. Therefore, their life expectancy is 80.1 years. A person 70 years old has a life expectancy of $70 + 15.5 = 85.5$ years. The person 70 years old has a longer life expectancy because that person has already survived the risk that a 20-year-old must endure between the ages of 20 and 70.

$[-10, 90, 10]$ by $[0, 80, 10]$

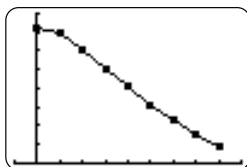


Figure 73

75. (a) From 1981 to 1985 the average rate of change is $\frac{f(1985) - f(1981)}{1985 - 1981} = \frac{6948 - 128}{4} = 1705$.
- From 1985 to 1990 the average rate of change is $\frac{f(1990) - f(1985)}{1990 - 1985} = \frac{31,269 - 6948}{5} = 4864.2$.
- From 1990 to 1995 the average rate of change is $\frac{f(1995) - f(1990)}{1995 - 1990} = \frac{45,765 - 31,269}{5} \approx 2899.2$.
- From 1995 to 1997 the average rate of change is $\frac{f(1997) - f(1995)}{1997 - 1995} = \frac{14,338 - 45,765}{2} = -15,713.5$.
- (b) From 1981 to 1985, the number of AIDS deaths increased by 1705 deaths per year. Other results can be interpreted similarly.
- (b) The temperature was rising the fastest on average from 12:00 to 1:00 and from 3:15 to 4:30. During these times it was rising at an average rate of 4°F per hour.

Chapter 1 Review Exercises

- -2 is an integer, rational number, and real number. $\frac{1}{2}$ is both a rational and a real number. 0 is an integer, rational number, and real number. 1.23 is both a rational and a real number. $\sqrt{7}$ is a real number. $\sqrt{16} = 4$ is a natural number, integer, rational number, and real number.
- $1,891,000 = 1.891 \times 10^6$
- $0.0000439 = 4.39 \times 10^{-5}$
- $1.52 \times 10^4 = 15,200$

9. (a) $\sqrt[3]{1.2} + \pi^3 \approx 32.07$ (b) $\frac{3.2 + 5.7}{7.9 - 4.5} \approx 2.62$
 (b) $\sqrt{5^2 + 2.1} \approx 5.21$ (d) $1.2(6.3)^2 + \frac{3.2}{\pi - 1} \approx 49.12$
11. $\frac{2.28 \times 10^8}{3 \times 10^5} = 760 \text{ seconds} = 12\frac{2}{3} \text{ minutes}$

13. The paint on the circular piece of paper can be thought of as a thin cylinder.

The volume of a cylinder is: $V = \pi r^2 h$. Substitute 0.25 in^3 in for V and 10 in. for r :

$$0.25 = \pi(10)^2 h \Rightarrow 0.25 = 100\pi h \Rightarrow h = \frac{0.25}{100\pi} \approx 7.96 \times 10^{-4}$$

The thickness of the paper is about 0.000796 inches.

15.

-23	-5	8	19	24
-----	----	---	----	----

(a) Maximum = 24; Minimum = -23

(b) Average = $\frac{-23 + (-5) + 8 + 19 + 24}{5} = 4.6$; Median = 8; Range = $24 - (-23) = 47$

17. (a) $D = \{-1, 4, 0, -5, 1\}$ and $R = \{-2, 6, -5, 3, 0\}$
 (b) $X_{\max} = 4, X_{\min} = -5, Y_{\max} = 6, Y_{\min} = -5$
 (c) & (d) The data is plotted in Figure 17.

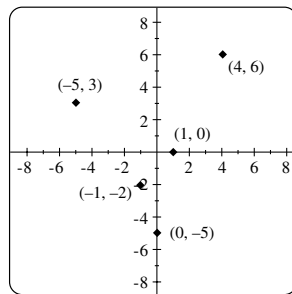


Figure 17

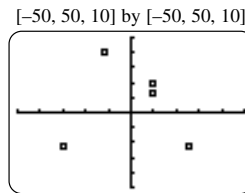


Figure 19

19. The relation $\{(10, 13), (-12, 40), (-30, -23), (25, -22), (10, 20)\}$ is plotted in Figure 19. It is not a function since both $(10, 13)$ and $(10, 20)$ are contained in the set. Notice that these points are lined up vertically.
21. $d = \sqrt{(-3 - 5)^2 + (2 - (-4))^2} = \sqrt{(-8)^2 + 6^2} = \sqrt{64 + 36} = \sqrt{100} = 10$
23. (a) The number of Catholic schools has declined.
 (b) From 1960 to 1970, $m = \frac{2.0 - 2.4}{10} = \frac{-0.4}{10} = -0.04$; the number of Catholic schools decreased on average by 40 schools per year from 1960 to 1970 (the units are in thousands).
 From 1970 to 1980, $m = \frac{1.5 - 2.0}{10} = \frac{-0.5}{10} = -0.05$; the number of Catholic schools decreased on average by 50 schools per year from 1970 to 1980 (the units are in thousands).
 From 1980 to 1990, $m = \frac{1.3 - 1.5}{10} = \frac{-0.2}{10} = -0.02$; the number of Catholic schools decreased on average by 20 schools per year from 1980 to 1990 (the units are in thousands).
 From 1990 to 1998, $m = \frac{1.22 - 1.3}{8} = \frac{-0.1}{8} = -0.01$; the number of Catholic schools decreased on average by 10 schools per year from 1990 to 1998 (the units are in thousands).

25. Symbolic: $f(x) = 16x$.

Numerical: Table f starting at $x = 0$, incrementing by 25. See Figure 25a.

Graphical: Graph $Y_1 = 16X$ in $[0, 100, 10]$ by $[0, 1800, 300]$. See Figure 25b.

x	0	25	50	75	100
$f(x)$	0	400	800	1200	1600

Figure 25a

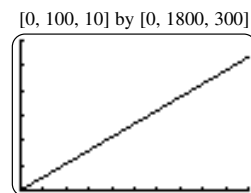


Figure 25b

27. (a) $f(-3) = 2$ and $f(1) = -2$

(b) $f(x) = 0$ when $x = -1$ and when $x = 3$

29. $f(x) = \sqrt[3]{x} \Rightarrow f(-8) = -2$ and $f(1) = 1$. The domain of f is all real numbers.

31. $f(x) = \frac{1}{x^2 - 4} \Rightarrow f(-3) = \frac{1}{(-3)^2 - 4} = \frac{1}{5}$ and $f(a + 1) = \frac{1}{(a^2 + 2a + 1) - 4} = \frac{1}{a^2 + 2a - 3}$

The domain of f is $D = \{x \mid x \neq \pm 2\}$.

33. No, since $x = y^2$ is not a function. An input of $x = 1$ produces outputs of $y = \pm 1$.

35. Since any vertical line intersects the graph of f at most once, it is a function.

37. $m = \frac{4 - 7}{3 - (-1)} = -\frac{3}{4}$

39. $m = \frac{4 - 4}{-2 - 8} = \frac{0}{-10} = 0$

41. $f(x) = 8 - 3x$ represents a linear function.

43. $f(x) = |x + 2|$ represents a nonlinear function.

45. $f(x) = 6$ represents a constant (and linear) function.

47. See Figure 47.

49. (a) Plot the points $(0, 100)$, $(1, 10)$, $(2, 6)$, $(3, 3)$, and $(4, 2)$ and make a line graph. See Figure 49. The data decreases rapidly at first. This means that a large number of eggs never develop into mature adults.

(b) Since any vertical line could intersect the graph at most once, this graph could represent a function.

(c) From 0 to 1, $\frac{10 - 100}{1 - 0} = -90$; from 1 to 2, $\frac{6 - 10}{2 - 1} = -4$; from 3 to 2, $\frac{3 - 6}{3 - 2} = -3$;

from 3 to 4, $\frac{2 - 3}{4 - 3} = -1$; during the first year, the population of sparrows decreased, on average, by 90 birds. The other average rates of change can be interpreted similarly.

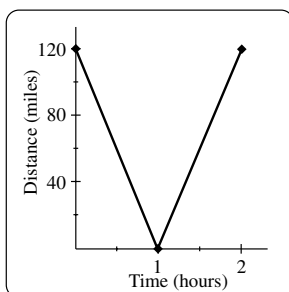


Figure 47

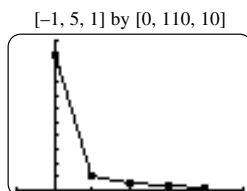


Figure 49

51. Linear, but not constant since the y -values decrease 8 units for every 2-unit increase in x .